Exam. Code : 103205
Subject Code : 1221

B.A./B.Sc. 5th Semester

MATHEMATICS Paper—I (Dynamics)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt any ***five*** questions in all choosing at least ***two*** from each section. All questions carry equal marks.

SECTION—A

1. Two cars start off the race with velocity u and v and travel in a straight line with uniform acceleration s and t. The cars reach the destination at the same time, Prove that the length of the course is $\frac{2(u-v)(ut-vs)}{(s-t)^2}$. 10

2. A point moving with uniform acceleration in a straight line describes equal distances in time t_1, t_2 and t_3, show that :

$$\frac{1}{t_1}-\frac{1}{t_2}+\frac{1}{t_3}=\frac{1}{t_1+t_2+t_3}.$$ 10

3. Two smooth inclined planes of equal heights and inclinations α and β are placed back to back. Masses m_1 and m_2 resting on them are connected by a light inextensible string over a smooth pulley fixed at common vertex of the two planes. If the system is free to move, discuss the motion. 10

4. A particle of mass m is acted upon by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance 'a' from the origin, show that it will reach the origin after time $\frac{\pi}{4\sqrt{\mu}}$. 10

5. (a) A particle moving with S.H.M. of period 30 sec travel 15 cm from the position of rest in 5 sec. Find the amplitude, the maximum velocity and velocity at the end of 5 sec.

   (b) A particle describes S.H.M. between two points $x = m$ and $x = -m$, the centre being at the origin. Find the relation between the velocity v, m and x if the maximum acceleration is 1. 5,5

SECTION—B

6. Define a projectile. Find the latus rectum, the vertex, the focus and the height of the directrix of the parabola traced by a projectile. 10

7. Let a particle is sliding down the convex side of a smooth vertical circle under gravity. If the initial velocity is that due to a free fall to the starting point from a height h above the centre, show that it will fly off the circle when at a height $\frac{2}{3}h$ above the centre. 10

8. A particle moves in a plane under a force which is always directed towards a fixed point in the plane. Obtain the differential equation of the orbit. 10

9. (a) Define work and power. Discuss F.P.S. and M.K.S. system of units of work and power.

   (b) A particle of mass m falls from rest at a height h above the ground. Show that throughout the motion, the sum of kinetic and potential energies is constant. 4,6

10. An elastic string of natural length l is extended by an amount a, when it supports a mass M at rest, and is extended by an amount b when it is rotating as a conical pendulum, carrying a particle of the same mass, with angular velocity w, prove that :

$$gb = w^2a(l + b).$$ 10